

# The finite element analysis of AZ91 extrusion based on Arrhenius constitutive relation and NC & L fracture criterion<sup>1</sup>

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**Abstract.** The Arrhenius constitutive equation of plastic deformation is achieved through the high-temperature compression experiment of AZ91 magnesium alloy. The critical value of fracture factor C of AZ91 magnesium alloy based on the Normalized Cockcroft & Latham fracture criterion is solved through a high-temperature tensile test. According to the constitutive equation and C, the extrusion of AZ91 plate is simulated by Deform-3D. The results show that the initial extrusion process with the temperature of 380 °C and extrusion speed of 0.8 mm/s is feasible, and the crack is predicted under the extrusion speed of 1.7 mm/s and the temperature of 380 °C.

**Key words.** AZ91 magnesium alloy, constitutive equation, fracture factor, finite element analysis..

## 1. Introduction

As a new "lightweight" engineering material, Magnesium alloy has a great prospect in energy conservation and environmental protection .Because of the close-packed hexagonal structure, which has less slip system, and hard plastic processing at room temperature, casting product is dominant in the use of magnesium alloy. Due to the superior comprehensive mechanical properties of deformation product compared with casting product, wide application and lower cost for industrialization, the research on the plastic deformation of magnesium alloys is of great significance.

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Many researches about the construction of the constitutive equation of magnesium alloy using Arrhenius equation have been done, and the description about high temperature mechanical properties of magnesium alloy is comparatively more accurate. The ZK60 Arrhenius constitutive equation is constructed by ductility test by Wang Fang et al [1]; The ZK31 Arrhenius constitutive equation is constructed by Wang Zhongtang et al [2]; The ZK81 Arrhenius constitutive equation is constructed by Cao Fenghong et al [3]; The AZ91D Arrhenius constitutive equation is constructed by Yu Sirong et al [4]. Due to different strengthening elements added in magnesium alloys, the thermoplastic will show much difference. The corresponding thermoplastic should be tested of particular brand of magnesium alloy to obtain the accurate data to build a suitable constitutive relation.

The Normalized C & L fracture criterion is applied widely in the mathematical model about the anticipation of material fracture. The critical damage value in high temperature environment of 9Cr1Mo is solved by Liu Yutong [5] using Normalized C & L fracture criterion; The crack distribution in the process of angular extrusion of 6061Al is analyzed by combining the critical damage value solved by the Normalized C & L fracture criterion with two-dimensional finite element simulation by Chen Wenjie et al [6]. Due to the difference of material composition, the critical damage value is different. The critical damage value of AZ91 is solved according to the test data, and the crack distribution in the plate extrusion process is judged by three-dimensional finite element simulation to guide the actual processing.

The Arrhenius constitutive equation of plastic deformation of AZ91 magnesium alloy is achieved through the compression experiment in different high-temperature and strain rate. The critical value of fracture factor C of AZ91 magnesium alloy based on the Normalized Cockcroft & Latham fracture criterion is solved by the high-temperature tensile test. According to the constitutive equation and C, the crack is predicted by simulating the extrusion of AZ91 plate with Deform-3D software.

## 2. Experimental procedure

### *2.1. High-temperature compression experiment*

The scheme of high-temperature compression experiment of AZ91 is shown in Table 1. Before compression, the homogenizing annealing is done at the temperature of 420 °C and remained 12 hours.

### *2.2. High-temperature tensile experiment*

The scheme of the high-temperature tensile experiment is shown in Table 2, and the tensile specimen size is shown in Fig. 1. Before tension, the homogenizing annealing is done at the temperature of 420 °C and held air cooling after 12 hours' remaining.

Table 1. Experimental scheme

Specimen size	$\varphi 10 \text{ mm} \times 15 \text{ mm}$
Heating rate	$10 \text{ }^\circ\text{C/s}$
Holding time	3 min
Compression temperature	$350 \text{ }^\circ\text{C}$ $400 \text{ }^\circ\text{C}$
Strain rate	$0.01 \text{ s}^{-1}$ $0.1 \text{ s}^{-1}$ $1 \text{ s}^{-1}$
Deformation degree	True strain 0.6
Model of machine	Gleeble3500

Table 2. Experimental scheme

Heating rate	$10 \text{ }^\circ\text{C/s}$
Holding time	3 min
Tensile temperature	$380 \text{ }^\circ\text{C}$
Strain rate	$0.01 \text{ s}^{-1}$
Model of machine	Gleeble3500

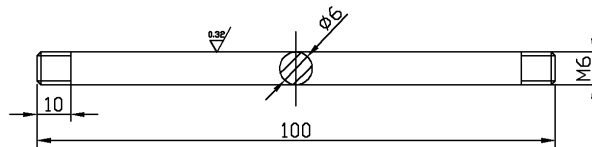


Fig. 1. Tensile specimen

Table 3. Analysis scheme

Extrusion speed	1.7 mm/s 0.8 mm/s
Temperature	380 °C
Extrusion ratio	17.54
Specimen size	$\varphi$ 100 mm $\times$ 50 mm cylindrical
Mesh number	35226
Extrusion size	wide 75 mm $\times$ thick 6 mm

### 2.3. Finite element analysis

The extrusion of AZ91 plate is simulated by Deform 3-D, and the analysis scheme is shown in Table 3. During the whole simulation process, the Sparse solver and Newton-Raphson iterative method are used.

## 3. Results and discussion

### 3.1. Arrhenius constitutive equation based on high-temperature compression experiment

The plastic processing of magnesium alloy should be under high temperature. There is a heat activation process under high temperature with the characteristic that the strain rate is controlled by it which follows the Arrhenius formula [7]

$$\dot{\varepsilon}' = \dot{\varepsilon}'_0 \exp\left(-\frac{Q}{RT}\right). \quad (1)$$

$Q$  means activation energy, kJ/mol;  $R$  means gas constant,  $8.314 \text{ J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$ ;  $K$  means Kelvin. The stress and strain rate meet the following exponential relationship during the plastic process with low stress

$$\dot{\varepsilon}' = A_1 \sigma^m, \quad (2)$$

$A_1$  and  $m$  are constant irrelevant to the temperature.

The stress and strain rate meets the following power exponent relationship during the plastic processing with high stress

$$\dot{\varepsilon}' = A_2 \exp(\beta\sigma). \quad (3)$$

According to (1), Sellars and Tegart [8] put forward the modifier Arrhenius for-

mula in form of hyperbolic sine

$$\epsilon' = A[\sinh(\alpha\sigma)]^n \exp\left(-\frac{Q}{RT}\right). \tag{4}$$

Formula (4) is approximate to (2) during the plastic processing with low stress, and approximate to (3) with high stress, so it can be used for the whole stress range. The constant  $\alpha$ ,  $\beta$  and  $m$  meet the following formula

$$\alpha = \frac{\beta}{m}. \tag{5}$$

According to the result of high-temperature compression experiment, the unknown parameters in (4) can be solved, then the Arrhenius constitutive equation. The curve about true strain and stress in high-temperature compression experiment is shown in Fig. 2.

McQueen [9] pointed out that the steady stress is chosen for the material with easy dynamic recovery in the study of the constitutive equation in high-temperature deformation, and the peak stress is usually chosen for the material with easy dynamic recrystallization. Formula (2), (3) and (4) are modified as follows

$$\ln \epsilon' = m \ln \sigma_p + \ln A_1, \tag{6}$$

$$\ln \epsilon' = \beta \sigma_p + \ln A_2, \tag{7}$$

$$\ln \epsilon' = n \ln[\sinh(\alpha\sigma_p)] + \ln A - \frac{Q}{RT}, \tag{8}$$

$$\ln[\sinh(\alpha\sigma_p)] = \frac{Q}{nRT} - \frac{\ln A}{n} + \frac{\ln \epsilon'}{n}, \tag{9}$$

$$\ln B = n \ln[\sinh(\alpha\sigma_p)] + \ln A. \tag{10}$$

According to the result of high-temperature compression experiment, formula (6) and (10) are fitted by Matlab. The unknown parameters in Arrhenius formula can be calculated based on the slope of the fitting line in Fig. 3, and the result is shown in Table 4.

Table 4. Result of unknown parameters in Arrhenius formula

$m$	$\beta$	$\alpha$	$n$	$Q$	$A$
6.59	0.0924	0.014	4.95	167635.17	$2.654e + 012$

The Arrhenius constitutive equation in high-temperature plastic deformation of

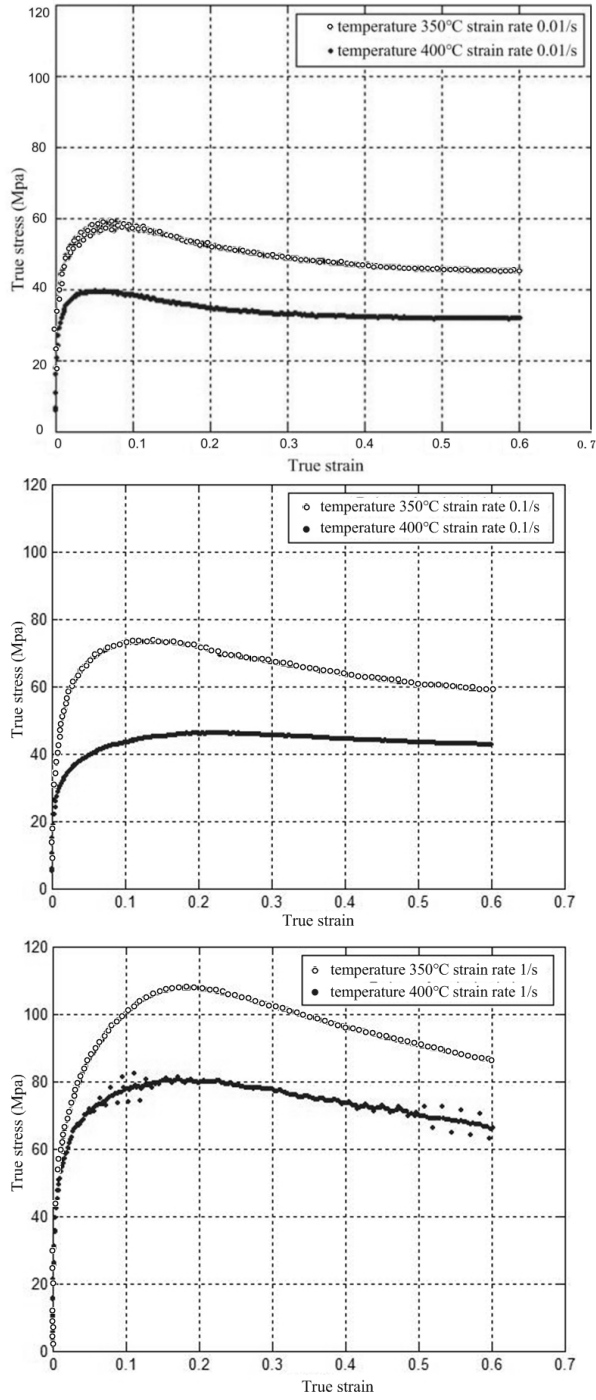


Fig. 2. Curve about true strain and stress

AZ91is as follows

$$\varepsilon' = 2.654 \times 10^{12} [\sinh(0.014\sigma)]^{4.95} \exp\left(-\frac{167635.17}{8.314T}\right). \tag{11}$$

**3.2. The critical value of fracture factor based on high-temperature tensile experiment**

The expression of Normalized Cockcroft & Latham fracture criterion is as follows

$$\int_0^{\bar{\varepsilon}_f} \left(\frac{\sigma_1}{\bar{\sigma}}\right) d\bar{\varepsilon} = C, \tag{12}$$

where  $\bar{\varepsilon}_f$  means equivalent fracture strain;  $\sigma_1$  means the maximum true stress along the tensile direction;  $\bar{\sigma}$  means the equivalent stress along the tensile direction;  $\bar{\varepsilon}$  means the equivalent strain;  $C$  means the fracture factor that shows the ability of resistance to fracture in specific high temperature and strain rate, and the higher the value, the stronger ability to resist fracture the material has [10].

The strain and stress along the tensile direction play a main role to the fracture during tension. Considering the difficulty and calculation, the  $\bar{\sigma}$  is replaced by engineering tensile strength  $\sigma_b$ ; the  $\bar{\varepsilon}$  is replaced by the true strain  $\varepsilon_1$  along the tensile direction; the  $\bar{\varepsilon}_f$  is replaced by the true strain  $\varepsilon_f$  at fracture. (12) can be modified into the following formula

$$\int_0^{\varepsilon_f} \left(\frac{\sigma_1}{\sigma_b}\right) d\varepsilon_1 = C. \tag{13}$$

The result of formula (12) is as follows

$$C = \int_0^{\varepsilon_f} \left(\frac{\sigma_1}{\sigma_b}\right) d\varepsilon_1 = \frac{\sigma_1 \varepsilon_f}{\sigma_b}. \tag{14}$$

The  $\sigma_1$  and  $\varepsilon_f$  in formula (14) can be obtained by the curve about true stress and strain in high-temperature tensile experiment, and  $\sigma_b$  is equal to the ratio of maximum tensile force and original cross-sectional area  $S$ .

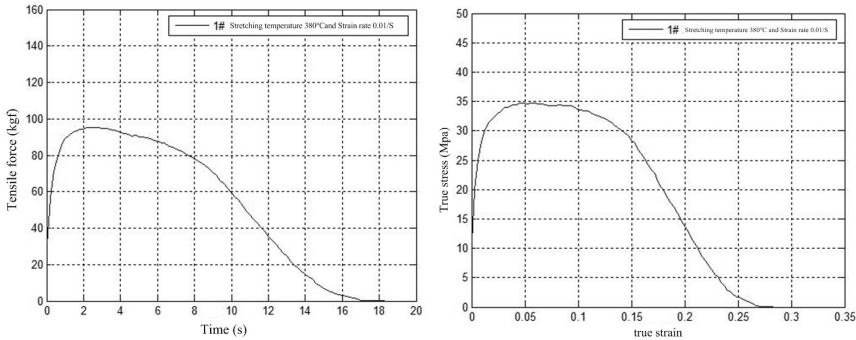


Fig. 3. Curves of high-temperature tensile experiment

According to the curve in Fig. 3, the following parameters in Table 5 are achieved.

Table 5. Experiment parameters

$\sigma_1$	$\varepsilon_f$	$F_{\max}$
34.77 Mpa	0.28426	95.40 kgf

According to the cross-sectional area of the tensile specimen,  $\sigma_p$  is evaluated to 33.07 Mpa, and the critical value of fracture factor  $C$  of AZ91 magnesium alloy under the temperature of 380 centigrade and the strain rate of 0.01/s is 0.3.

### 3.3. Simulation analysis of extrusion

Taking formula (12) as the constitutive relation of finite element analysis of AZ91 extrusion, and  $C = 0.3$  as the criterion of crack initiation, Deform 3-D is applied to the simulation of extrusion process of AZ91, and the distribution pattern and isosurface of fracture factor are shown in Fig. 4: the extrusion speed is 1.7 mm/s in Fig. 4(a),(b), 0.8 mm/s in Fig. 4(c), (d).

Fig. 4(a) shows that the highest of the fracture factor is near 0.3; the tendency of surface crack is obvious. As shown in Fig. 4, the red isosurface of fracture factor extends to the inside of extrusion, which means the crack is expanding to internal. Fig. 4(c) and (d) show that all the fracture factor is below 0.3; the tendency of crack is not obvious.

The practical extrusions are shown in Fig. 5. The crack in the side of extrusion is obviously in Fig. 5(a); The extrusion in Fig. 5(b) has a better quality without the crack. The results of practical extrusion and simulation are identical with each other.

## 4. Conclusion

1. The constitutive equation in high-temperature plastic deformation of AZ91 magnesium alloy is as follows,  $\varepsilon' = 2.654 \times 10^{12} [\sinh(0.014\sigma)]^{4.95} \exp(-\frac{167635.17}{8.314T})$ . The temperature range is 350 °C to 400 °C, and the range of strain rate is 0.01 s<sup>-1</sup> to 1 s<sup>-1</sup>.
2. The critical value of fracture factor  $C$  is evaluated to 0.3 under tensile experiment with the temperature of 380 °C and the strain rate of 0.01 s<sup>-1</sup> of AZ91 magnesium alloy.
3. The simulation of the extrusion process of AZ91 by Deform 3-D verifies the feasibility of the initial extrusion process with the temperature of 380 °C and the speed of 0.8 mm/s. The crack under the condition of the temperature of 380 °C and the speed of 1.7 mm/s is predicted.



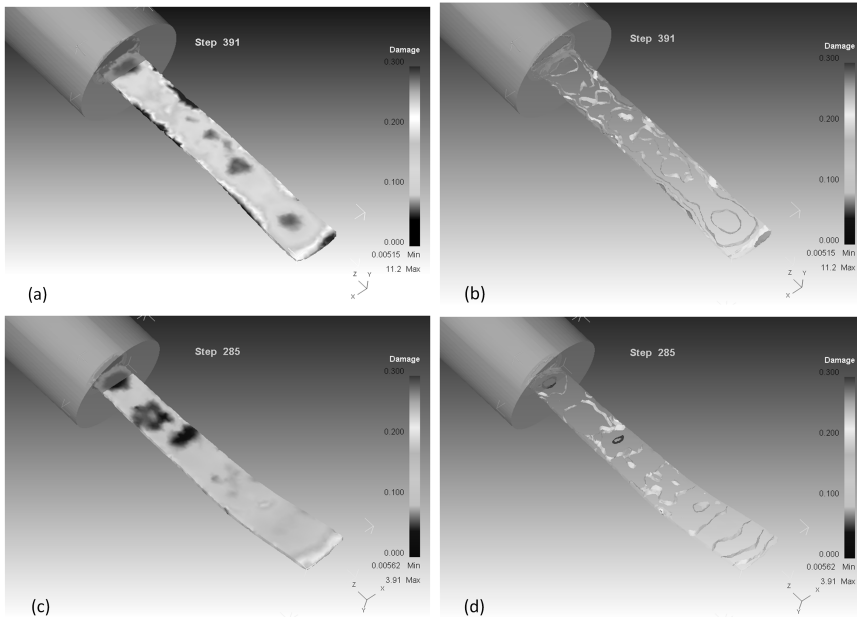


Fig. 4. Isosurface of fracture factor

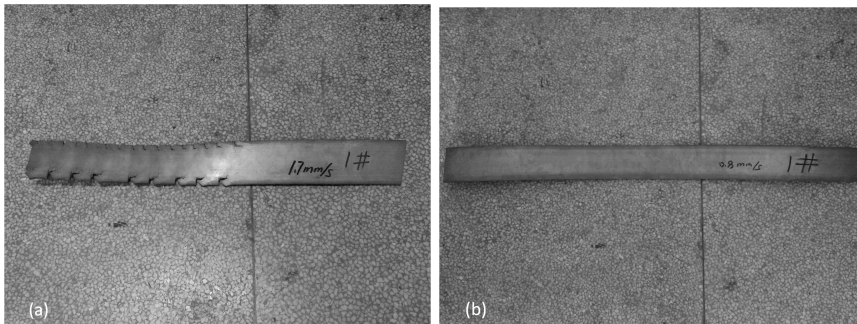


Fig. 5. Practical extrusion effects

4. Because many literature related to Normalized C & L fracture criterion are about the forecasting of steel fracture, the fracture criterion used for Al or Mg need to be explored, such as the construction of the fracture model, experimental model and mathematical model.
5. According to the future development trend, the research of the Anti-Matter, Dark-Matter, electromagnetic field, vibration, light can reveal the laws of substance, energy and the universe to produce more advanced materials.

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